MATHEMATICS

SECTION 1 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If **ONLY** the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : -1 In all other cases.

1. Let $\mathbb{R}$ denote the set of all real numbers. Let $a_i, b_i \in \mathbb{R}$ for $i\{1,2,3\}$.

Define the function $f:\mathbb{R}\to\mathbb{R}, g:\mathbb{R}\to\mathbb{R}$, and $h:\mathbb{R}\to\mathbb{R}$ by.

$f(x)=a_1+10x+a_2x^2+a_3x^3+x^4$,

$g(x)=b_1+3x+b_2x^2+b_3x^3+x^4$,

$h(x)=f(x+1)-g(x+2)$

If $f(x)\neq g(x)$ for every $x\in\mathbb{R}$, then the coefficient of x^3 in $h(x)$ is

(A) 8 (B) 2 (C) -4 (D) -6

Ans. **(C)**

Sol. $h(x)=f(x+1)-g(x+2)$

$a_1+10(x+1)+a_2(x+1)^2+a_3(x+1)^3+(x+1)^4-(b_1+3(x+2)+b_2(x+2)^2+b_3(x+2)^3+(x+2)^4$

Coefficient of $x^3=a_3+4-(b_3+8)$

$=a_3-b_3-4$

& $f(x)\neq g(x)$

$a_1+10x+a_2x^2+a_3x^3+x^4\neq b_1+3x+b_2x^2+b_3x^3+x^4$

$(a_3-b_3)x^3+(a_2-b_2)x^2+7x+a_1-b_1\neq 0$

As in the question given $f(x)\neq g(x)$. That means this equation does not have real roots, but a cubic equation with real coefficient always have at least one real root. To satisfy the given condition in this question. This should be quadratic with non real roots. So $a_3-b_3=0$

Then coefficient of $(x)^3$ in $h(x)$ is -4 .

2. Three students S_1, S_2 and S_3 are given a problem to solve. Consider the following events.

U : At least one S_1, S_2, and S_3 can solve the problem

$V : S_1$ can solve the problem, given that neither S_2 nor S_3 can solve the problem

$W : S_2$ can solve the problem and S_3 cannot solve the problem,

$T : S_3$ can solve the problem

For any event E, let $P(E)$ denote the probability of E. If

$P(U)=\frac{1}{2}, P(V)=\frac{1}{10}$, and $P(W)=\frac{1}{12}$,

then $P(T)$ is equal to

(A) $\frac{13}{36}$ (B) $\frac{1}{3}$ (C) $\frac{19}{60}$ (D) $\frac{1}{4}$

Ans. **(A)**

Sol. Let $P(\text{ Only } S_1)=x$

$$P(S_1 \cup S_2 \cup S_3)=\frac{1}{2}$$

$$P\left(S_1 | (S_2 \cup S_3)^c\right)=\frac{x}{x+0.5}=0.1 \Rightarrow x=\frac{5}{90}=\frac{1}{18}$$

$$P\left(S_2 | S_3^c\right)=\frac{1}{12}$$

$$P(T)=\frac{1}{2}-\frac{1}{18}-\frac{1}{12}=\frac{18-2-3}{36}=\frac{13}{36}$$

3. Let $\mathbb{R}$ denote the set of all real numbers. Define the function $f:\mathbb{R}\to\mathbb{R}$ by

$$f(x)=\begin{cases} 2-2x^2-x^2\sin\frac{1}{x} & \text{if } x\neq 0 \\ 2 & \text{if } x=0 \end{cases}$$

Then which one of the following statements is TRUE ?

(A) The function f is NOT differentiable at $x=0$

(B) There is a positive real number δ, such that f is a decreasing function on the interval $(0,\delta)$

(C) For any positive real number δ, the function f is NOT an increasing function on the interval $(\delta, 0\ )$

(D) $x=0$ is a point of local minima of f

Ans. **(C)**

Sol. $$f(x)=\begin{cases} 2-2x^2-x^2\sin\left(\frac{1}{x}\right) & x\neq 0 \\ 2 & x=0 \end{cases}$$

$$\lim_{x\to 0}\frac{f(x)-f(0)}{x}=\lim_{x\to 0}-\left(2x+x\sin\left(\frac{1}{x}\right)\right)=0 \Rightarrow f \text{ is differentiable at } x=0$$

$$f'(x)=\begin{cases} -4x-2x\sin\left(\frac{1}{x}\right)+\cos\left(\frac{1}{x}\right) & x\neq 0 \\ 0 & x=0 \end{cases}$$

$$f'(x)=-2x\left\{2+\sin\left(\frac{1}{x}\right)\right\}+\cos\left(\frac{1}{x}\right)\quad x\neq 0.$$

Also at $x=0, f(x)$ has a maxima

$f'(x)$ cannot be same sign in $(0,\delta)$ or $(-\delta,0)$

So it cannot be monotonic.

4. Consider the matrix $P=\begin{pmatrix}2&0&0\\0&2&0\\0&0&3\end{pmatrix}$ Let the transpose of a matrix X be denoted by $X^{\top}$. Then the number of 3×3 invertible matrices Q with integer entries, such that $Q^{-1}=Q^{\top}$ and $PQ=QP$, is

(A) 32 (B) 8 (C) 16 (D) 24

Ans. **(C)**

Sol. $P=\begin{bmatrix}2&0&0\\0&2&0\\0&0&3\end{bmatrix}$

Let $Q=\begin{bmatrix}a&b&c\\d&e&f\\g&h&i\end{bmatrix}$

Now $QP=\begin{bmatrix}2a&2b&3c\\2d&2e&3f\\2g&2h&3i\end{bmatrix}$

$PQ=\begin{bmatrix}2a&2b&2c\\2d&2e&2f\\3g&3h&3i\end{bmatrix}$

As $PQ=QP$

By comparing $C=0$ & $g=0$ & $f=h=0$

So $Q=\begin{bmatrix}a&b&0\\d&e&0\\0&0&i\end{bmatrix}$

$Q=\begin{bmatrix}a&b&0\\d&e&0\\0&0&i\end{bmatrix}$

$Q^{\top}=\begin{bmatrix}a&d&0\\b&e&0\\0&0&i\end{bmatrix}$

If $Q^{-1}=Q^{\top}$

$Q-Q^{\top}=I$

$\begin{bmatrix}a&b&0\\d&e&0\\0&0&i\end{bmatrix}\begin{bmatrix}a&d&0\\b&e&0\\0&0&i\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

Then $a^2+b^2=1$ & $ad+be=0$

$d^2+e^2=1$

$i^2=1$

$i=+1$ or -1

As a,b,d,e,i are integers then

$a^2=1\ \&\ b^2=0, d=0, e^2=1$

$b^2=1\ \&\ a^2=0, e=0, d^2=1$

Total no. of ways $=16$

SECTION 2 (Maximum Marks: 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	: +4	**ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks	: +3	If all the four options are correct but **ONLY** three options are chosen;
Partial Marks	: +2	If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks	: +1	If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks	: 0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	: -2	In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY(A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and choosing any other combination of options will get -2 marks.

5. Let L_1 be the line intersection of the planes given by the equations $2x+3y+z=4$ and $x+2y+z=5$. Let L_2 be the line passing through the point $P(2,-1,3)$ and parallel to L_1. Let M denote the plane given by the equation $2x+y-2z=6$. Suppose that the line L_2 meets the plane M at the point Q. Let R be the foot of the perpendicular drawn from P to plane M. Then which of the following statements is (are) TRUE ?

(A) The length of the line segment PQ is $9\sqrt{3}$

(B) The length of the line segment QR is 15

(C) The area of ΔPQR is $\frac{3}{2}\sqrt{234}$

(D) The acute angle between the line segment PQ and PR is $\cos^{-1}\left(\frac{1}{2\sqrt{3}}\right)$

Ans. (A,C)

Sol. Vector along $L_1 = (2\hat{i}+3\hat{j}+\hat{k})\times(\hat{i}+2\hat{j}+\hat{k})$

$= \hat{i} - \hat{j} + \hat{k}$

Equation of $L_2 : \frac{x-2}{1} = \frac{y+1}{-1} = \frac{z-3}{1}$

$Q \equiv (\lambda+2, -\lambda-1, \lambda+3)$

Q lies on M,

$2(\lambda+2)+(-\lambda-1)-2(\lambda+3)=6$

$\Rightarrow 2\lambda+4-\lambda-1-2\lambda-6=6$

$\Rightarrow \lambda=-9$

$Q \equiv (-7,8,-6)$

Let $R \equiv (x_1, y_1, z_1)$

$\frac{(x_1-2)}{2} = \frac{y_1+1}{1} = \frac{z_1-3}{-2} = -\frac{(2\times2-1-2\times3-6)}{9} = 1$

$x_1 \equiv 4,\ y_1 = 0,\ z_1 = 1$.

$\therefore R \equiv (4,0,1)$.

$PQ = \sqrt{9^2+9^2+9^2} = 9\sqrt{3}$

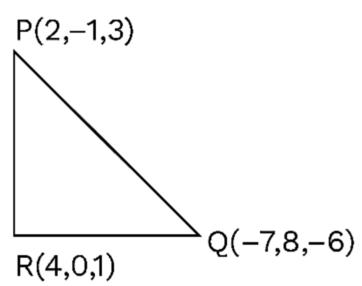
$QR = \sqrt{(-11)^2+8^2+7^2} = \sqrt{234}$

area of $\triangle PQR = \frac{1}{2}\left|\overrightarrow{PR}\right|\times\left|\overrightarrow{QR}\right| = \frac{3}{2}\sqrt{234}$

$\cos\theta = \left|\frac{(2\hat{i}+\hat{j}-2\hat{k})\cdot(\hat{i}-\hat{j}+\hat{k})}{|2\hat{i}+\hat{j}-2\hat{k}|\ |\hat{i}-\hat{j}+\hat{k}|}\right| = \frac{1}{3\sqrt{3}}$

$\therefore \theta = \cos^{-1}\left(\frac{1}{3\sqrt{3}}\right)$

$P(2,-1,3)$



6. Let $\mathbb{N}$ denote the set of all natural numbers, and $\mathbb{Z}$ denote the set of all integers. Consider the functions $f:\mathbb{N}\to\mathbb{Z}$ and $g:\mathbb{Z}\to\mathbb{N}$ defined by

$$f(n)=\begin{cases}(n+1)/2 & \text{if } n \text{ is odd}\\ (4-n)/2 & \text{if } n \text{ is even}\end{cases} \text{ and } g(n)=\begin{cases}3+2n & \text{if } n\geq 0\\ -2n & \text{if } n<0\end{cases}$$

Define $(g\circ f)(n)=g(f(n))$ for all $n\in\mathbb{N}$, and $(f\circ g)(n)=f(g(n))$ for all $n\in\mathbb{Z}$.

Then which of the following statements is (are) TRUE?

(A) gof is NOT one-one and gof is NOT onto

(B) fog is NOT one-one but fog is onto

(C) g is one-one and g is onto

(D) f is NOT one-one but f is onto

Ans. **(AD)**

Sol. $$\text{fog }(n)=\begin{cases}2+n & n\geq 0\\ 2+n & n<0\end{cases}$$ Also fog: $\mathbb{Z}\to\mathbb{Z}$.

$$\text{gog}(n)=\begin{cases}n+4 & n=\text{ odd.}\\ 7-n & n=2\\ n-4 & n=\text{ even}-\{2\}.\end{cases}$$ Also gof : $\mathbb{N}\to\mathbb{N}$.

Clearly gof is not one-one as. gof $(n)=5$ for both $n=1$ and $n=2$

Also gof is not onto as $\text{gof}(n)\neq 1$. For any value of n.

fog is both one-one and onto.

$g(x)$ can never be equal to one. Hence $g(x)$ is not onto

hence (c) is incorrect

f is many one as for both $n=2$ and $n=1$

the output is 1.

f is onto.

7. Let $\mathbb{R}$ denote the set of all real numbers. Let $z_1=1+2i$ and $z_2=3i$ be two complex numbers, where $i=\sqrt{-1}$. Let $S=\{(x,y)\in\mathbb{R}\times\mathbb{R}:|x+iy-z_1|=2|x+iy-z_2|\}$.

Then which of the following statements is (are) TRUE?

(A) S is a circle with centre $\left(\frac{-1}{3},\frac{10}{3}\right)$

(B) S is a circle with centre $\left(\frac{1}{3},\frac{8}{3}\right)$

(C) S is a circle with radius $\frac{\sqrt{2}}{3}$

(D) S is a circle with radius $\frac{2\sqrt{2}}{3}$

Ans. **(AD)**

Sol. S represents the curve

$\left|\frac{z-z_1}{z-z_2}\right|=2$ which is a circle

centre of circle $=\left(\frac{-1}{3},\frac{10}{3}\right)$

radius of circle $=\frac{2|z_1-z_2|}{2^2-1}$

$=\frac{2\sqrt{2}}{3}$

SECTION 3 (Maximum Marks: 24)

- This section contains SIX (06) questions.
- The answer to each question is a NUMERICAL VALUE.
- For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, truncate/roundoff the value to TWO decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 ONLY if the correct numerical value is entered;

  Zero Marks : 0 In all other cases.

8. Let the set of all relations R on the set $\{a,b,c,d,e,f\}$, such that R is reflexive and symmetric, and R contains exactly 10 elements, be denoted by $\mathcal{S}$ is .
Then the number of elements in $\mathcal{S}$ is .

Ans. **105**

Sol. $\because R$ is reflexive it must contain ($a,a,(b,b),(c,c),(d,d),(e,e)$ and (f,f) i.e. 6 elements-

$\because R$ is also symmetric so it contains elements of form (x,y) and (y,x).

$\because$ R contains exactly 10 elements-

so it must contain 4 elements of form $(a_1,b_1)(b_1,a_1)\&(a_2,b_2),(b_2,a_2)$.

So No. of elements in $\mathcal{S} =$ No. of ways we can form 2 ordered pairs $(a_1,\ b_1),(a_2,\ b_2)$.

$= {}^6c_3\times 3+{}^6c_4\times 3$

$=105$

9. For any two points M and N in the X Y -plane, let $\overrightarrow{MN}$, denote the vector from M to N , and $\vec{0}$ denote the zero vector. Let P,Q and R be three distinct points in the XY -plane. Let S be a point inside the triangle $\triangle PQR$ such that

$\overrightarrow{SP}+5\overrightarrow{SQ}+6\overrightarrow{SR}=\vec{0}$

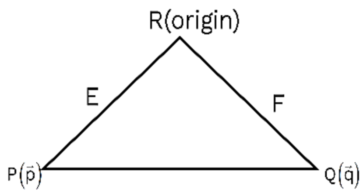
Let E and F be the mid-points of the sides PR and QR, respectively. Then the value of

$$\frac{\text{length of the line segment EF}}{\text{length of the line segment ES}}$$

is.

Ans. **1.2**

Sol. With R as origin let $\vec{p}.\vec{v}$ of P and Q be $\vec{p}\,\&\,\vec{q}$ respectively.



Now $\vec{s}=\dfrac{\vec{p}+5\vec{q}}{12}$

$\overrightarrow{EF}=\dfrac{\vec{q}-\vec{p}}{2},\ \overrightarrow{ES}=\dfrac{\vec{p}+5\vec{q}}{12}-\dfrac{\vec{p}}{2}=\dfrac{5(\vec{q}-\vec{p})}{12}$

Hence $\dfrac{EF}{ES}=\dfrac{12}{5\times 2}=1.2$

10. Let S be the set of all seven-digit numbers that can be formed using the digits 0,1 and 2. For example, 2210222 is in S, but 0210222 is NOT in S. Then the number of elements x in S such that at least one of the digits 0 and 1 appears exactly twice in x, is equal to.

Ans. **762**

Sol. $A_0 : 0$ appears twice

$A_1 : 1$ appears twice

$n(A_0) = {}^6C_2.2^5 = 480$

$n(A) = \boxed{1}\square\square\square\square\square\square + \boxed{2}\square\square\square\square\square\square$

${}^6C_1.2^5 + {}^6C_2.2^4 = 432$

$n(A_0 \cap A_1) = \frac{7!}{2!\cdot 2!\cdot 3!} - \frac{6!}{2!\cdot 3!} = \frac{720\times 7}{24} - \frac{720}{12}$

$= 210 - 60 = 150$

Hence $n(A_0 \cup A_1) = 480 + 432 - 150 = 762$

11. Let α and β be the real numbers such that $\lim_{x\to 0}\frac{1}{x^3}\left(\frac{\alpha}{2}\int_0^x \frac{1}{1-t^2}dt + \beta x\cos x\right) = 2$

Then the value of $\alpha + \beta$ is.

Ans. **1.2**

Sol. $\lim_{x\to 0}\frac{\frac{\alpha}{2}\int_0^x \frac{dt}{1-t^2} + \beta x\cos x}{x^3} = 2$

$= \lim_{x\to 0}\frac{\frac{\alpha}{2(1-x^2)} - \beta x\sin x + \beta\cos x}{3x^2} = 2$

$\Rightarrow \lim_{x\to 0}\frac{\frac{\alpha}{2} + \beta(1-x^2)(\cos x - x\sin x)}{3x^2(1-x^2)} = 2$.

$\Rightarrow \lim_{x\to 0}\frac{\left(\frac{\alpha}{2} + \beta(1-x^2)\left(1 - \frac{x^2}{2} - x^2 \ldots\ldots\right)\right)}{3x^2(1-x^2)} = 2$

$\Rightarrow \lim_{x\to 0}\frac{\left(\frac{\alpha}{2} + \beta\right) - \frac{5\beta}{2}x^2 - \cdots}{3x^2(1-x^2)} = 2$

So, $\alpha = -2\beta$ & $\beta = -\frac{6}{5}$ & $\alpha = \frac{12}{5}$

$\alpha + \beta = 6/5 = 1.2$

12. Let $\mathbb{R}$ denote the set of all real numbers. Let $f:\mathbb{R}\to\mathbb{R}$ be a function such that $f(x)>0$ for all $x\in\mathbb{R}$, and $f(x+y)=f(x)f(y)$ for all $x,y\in\mathbb{R}$.
Let the real numbers $a_1,a_2,\ldots,a_{50}$ be in an arithmetic progression. If $f(a_{31})=64f(a_{25})$, and $\sum_{i=1}^{50}f(a_i)=3(2^{25}+1)$ then the value of $\sum_{i=6}^{30}f(a_i)$ is

Ans. 96

Sol. $f(x+y)=f(x)f(y)$

$\Rightarrow f(a+d)=f(a)f(d).,f(a+2d)=f(a).(f(d))^2$

Hence $f(a+(r-1)d)=f(a)\cdot(f(d))^{r-1}$

Now

$f(a_{31})=64f(a_{25})$

$\Rightarrow f(a_1)\cdot(f(d))^{30}=64\cdot f(a_1)\cdot(f(d))^{24}\Rightarrow(f(d))^6=64\Rightarrow f(d)=2($ as $f(d)>0)$

Also $\sum_{i=1}^{50}f(a_i)=f(a_1)\left(1+f(d)+(f(d))^2+\cdot+(f(d))^{49}\right)$

$3(2^{25}+1)=f(a_1))(2^{50}-1)$

$\Rightarrow f(a_1)=\dfrac{3}{2^{25}-1}$

Now $f(a_6)+f(a_7)+\quad +f(a_{30})$

$=f(a_1)\cdot 2^5\{1+2+2^2+\cdots 2^{24}\}$

$=\dfrac{3}{2^{25}-1}\cdot 2^5\cdot(2^{25}-1)=96$

13. For all $x>0$, let $y_1(x),y_2(x)$, and $y_3(x)$ be the functions satisfying

$\dfrac{dy_1}{dx}-(\sin x)^2y_1=0,\ y_1(1)=5\quad \dfrac{dy_2}{dx}-(\cos x)^2y_2=0,\ y_2(1)=\dfrac{1}{3},\ \dfrac{dy_3}{dx}-\left(\dfrac{2-x^3}{x^3}\right)y_3=0,\ y_3(1)=\dfrac{3}{5e}$

respectively. Then $\lim_{x\to0^+}\dfrac{y_1(x)y_2(x)y_3(x)+2x}{e^{3x}\sin x}$ is equal to.

Ans. 2

Sol. $\int\dfrac{dy}{y}=\int\sin^2x\,dx$

$\ln y=\dfrac{x}{2}-\dfrac{\sin 2x}{4}+c$

$\ln 5=\dfrac{1}{2}-\dfrac{\sin 2x}{4}+c$

$y_1=e^{\left(\frac{x}{2}-\frac{\sin 2x}{4}\right)+c_1}$

$y_2=e^{\frac{x}{2}+\frac{\sin 2x}{4}+c_2}$

$\ln y_3=\int\left(\dfrac{2}{x^3}-1\right)dx$

$=-x^{-2}-x+c_3$

$y_3=e^{-\left(\frac{1}{x^2}+x\right)+c_3}$

$\lim_{x\to0^+}\dfrac{e^{c_1+c_2+c_3}\cdot e^{\left(x-x-\frac{1}{x^2}\right)}+2x}{e^{3x}\sin x}$

$\lim_{x\to0^+}\dfrac{k\cdot e^{\left(-\frac{1}{x^2}\right)}+2x}{e^{3x}\sin x}=\dfrac{2x}{\lim_{x\to0^+}e^{3x}\sin x}=\dfrac{2}{1}=2$

SECTION 4 (Maximum Marks: 12)

- This section contains THREE (03) Matching List Sets.
- Each set has ONE Multiple Choice Question.
- Each set has TWO lists: List-I and List-II.
- List-I has Four entries (P), (Q), (R) and (S) and List-II has Five entries (1), (2), (3), (4) and (5).
- FOUR options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 ONLY if the option corresponding to the correct combination is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : -1 In all other cases.

14. Consider the following frequency distribution :

Value	4	5	8	9	6	12	11
Frequency	5	F_1	F_2	2	1	1	3

Suppose that the sum of the frequencies is 19 and the median of this frequency distribution is 6 .

For the given frequency distribution, let α denote the mean deviation about the mean, β denote the mean deviation about the median, and σ^2 denote the variance.

Match each entry in List-I to the correct entry in List-II and choose the correct option.

List-I	**List-II**
(P) $7f_1 + 9f_2$ is equal to	(1) 146
(Q) 19α is equal to	(2) 47
(R) 19β is equal to	(3) 48
(S) $19\sigma^2$ is equal to	(4) 145
	(5) 55

(A) $(P)\rightarrow(5)(Q)\rightarrow(3)(R)\rightarrow(2)(S)\rightarrow(4)$

(B) $(P)\rightarrow(5)(Q)\rightarrow(2)(R)\rightarrow(3)(S)\rightarrow(1)$

(C) $(P)\rightarrow(5)(Q)\rightarrow(3)(R)\rightarrow(2)(S)\rightarrow(1)$

(D) $(P)\rightarrow(3)(Q)\rightarrow(2)(R)\rightarrow(5)(S)\rightarrow(4)$

Ans. **(C)**

Sol. Sum of frequencies $=12+f_1+f_2=19$

$f_1+f_2=7$

median to 10^{th} term i.e. '6'

$f_1=4;f_2=3$

(P) $7f_1+9f_2=7\times4+9\times3=28+27=55$

(Q)

x	Frequency	$\lvert x_i-x_m\rvert$	$f\lvert x_i-x_m\rvert$
4	5	3	15
5	4	2	8
6	1	1	1
8	3	1	3
9	2	2	4
11	3	4	12
12	1	5	5
			48

Mean $=\dfrac{4\times5+5\times4+6\times1+8\times3+9\times2+11\times3+12\times1}{19}=7$

$\alpha=\dfrac{48}{19}$

$19\alpha=48$

(R) mean deviation about mean

xi	Frequency	$\lvert x_i-x_m\rvert$	$f\lvert x_i-x_m\rvert$
4	5	2	10
5	4	1	4
6	1	0	0
8	3	2	6
9	2	3	6
11	3	5	15
12	1	6	6
			47

$\beta=\dfrac{47}{19}$ So $19\beta=47$

(S) $\sigma^2=\dfrac{\Sigma f_i x_i^2}{\Sigma f_i}-(7)^2=\dfrac{146}{19}$

$19\sigma^2=146$

15. Let R denote the set of all real numbers. For a real number x, let $[x]$ denote the greatest integer less than or equal to x. Let n denote a natural number.

Match each entry in List - I to the correct entry in List - II and choose the correct option.

List-I	List-II
(P) The minimum value of n for which the function $f(x)=\left[\frac{10x^3-45x^2+60x+35}{n}\right]$ is continuous on the interval $[1,2]$, is	(1) 8
(Q) The minimum value of n for which $g(x)=(2n^2-13n-15)(x^3+3x)$, $x\in\mathbb{R}$, is an increasing function on R, is	(2) 9
(R) The smallest natural number n Which is greater than 5, such that $x=3$ is a point of local minima of $h(x)=(x^2-9)^n(x^2+2x+3)$, is	(3) 5
(S) Number of $x_0\in\mathbb{R}$ such that $L(x)=\sum_{k=0}^{4}\left(\sin\lvert x-k\rvert+\cos\left\lvert x-k+\frac{1}{2}\right\rvert\right)$, $x\in\mathbb{R}$, is NOT differentiable at x_0, is	(4) 6
	(5) 10

$(A)(P)\to(1)(Q)\to(3)(R)\to(2)(S)\to(5)$

$(B)(P)\to(2)(Q)\to(1)(R)\to(4)(S)\to(3)$

$(C)(P)\to(5)(Q)\to(1)(R)\to(4)(S)\to(3)$

$(D)(P)\to(2)(Q)\to(3)(R)\to(1)(S)\to(5)$

Ans. **(B)**

Sol. (P) let

$\phi(x)=10x^3-45x^2+60x+35$

$\phi'(x)=30x^2-90x+60=30(x^2-3x+2)$

Range of $\phi(x)$ will be $[\phi(2),\phi(1)]$

$\phi(2)=80-180+120+35=55$

$\phi(1)=10-45+60+35=60$

range [55, 60]

$n_{min}=9$ for which this is a continuous

$P\to$ (2)

(Q) $g'(x)=(2n^2-13n-15)(3x^2+3)$

for this $2n^2-13n-15>0$

$n_{min}=8$

$Q\to(1)$

(R) $h'(x)=n(x^2-9)^{n-1}(x^2+2x+3)+(x^2-9)^n(2x+2)$

For minima $x=3$

$$=(x^2-9)^{n-1}\left[\underbrace{\left[n(x^2+2x+3)+2x+2\right]}_{\text{always Positive around } 3}\right]$$

$n-1$ must be odd $\Rightarrow n$ must be even.

$n_{min}=6$

$R\to(4)$

(S) $l(x)=\sin|x-4|+\sin|x-3|+\sin|x-2|+\sin|x-1|$

$+\sin|x|+\cos\left(x+\frac{1}{2}\right)+\cos\left|x-\frac{1}{2}\right|+\ldots\cos\left|x-\frac{7}{2}\right|$

Is non differentiable at 0,1,2,3,4.

(5) points

(S) $\to 3$

16. Let $\vec{w}=\hat{i}+\hat{j}-2k$, , and $\dot{u}$ and $\vec{v}$ be two vectors, such that $\dot{u}\times\dot{v}=\dot{w}$ and $\dot{v}\times\dot{w}=\dot{u}$. Let α,β,γ, and t be real numbers such that

$\vec{u}=\alpha\hat{i}+\beta\hat{j}+\gamma k$, $\quad -t\alpha+\beta+\gamma=0$, $\quad \alpha-t\beta+\gamma=0$, , and $\alpha+\beta-t\gamma=0$.

Match each entry in List-I to the correct entry in List-II and choose the correct option.

List-I	List – II
(P) $\|\vec{v}\|^2$ is equal to	(1) 0
(Q) If $\alpha=\sqrt{3}$, then is γ^2 equal to	(2) 1
(R) If $\alpha=\sqrt{3}$, then $(\beta+\gamma)^2$ is equal to	(3) 2
(S) If $\alpha=\sqrt{2}$, then $t+3$ is equal to	(4) 3
	(5) 5

(A) $(P)\to(2)(Q)\to(1)(R)\to(4)(S)\to(5)$

(B) $(P)\to(2)(Q)\to(4)(R)\to(3)(S)\to(5)$

(C) (P) $\to$ (2) (Q) $\to$ (1) (R) $\to$ (4) (S) $\to$ (3)

(D) $(P)\to(5)(Q)\to(4)(R)\to(1)(S)\to(3)$

Ans. **(Bonus)**

Sol. $\vec{w}=\hat{u}+\hat{j}-2\hat{k}$

$\vec{u}\times\vec{v}=\vec{w}$ & $\vec{v}\times\vec{w}=\vec{u}$

Note that $\vec{u},\vec{v},\vec{w}$ are mutually perpendicular vector

Hence $|\vec{u}||\vec{v}|=\sqrt{6}$ & $\sqrt{3}|\vec{v}|=|\vec{u}|$

$\Rightarrow|\vec{v}|^2=\sqrt{2}$

$\vec{u}\times\vec{v}=\vec{w}$

$\vec{v}\times(\vec{u}\times\vec{v})=\vec{v}\times\vec{w}=\vec{u}$

$(\vec{v})^2\cdot\vec{u}=\vec{u}=|\vec{v}|^2=1$

data inconsistent, so question should be given Bonus.