MATHEMATICS

SECTION 1 (Maximum Marks: 12)

- This section contains FOUR (04) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +3 If ONLY the correct option is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : -1 In all other cases.

1. Let x_0 be the real number such that $e^{x_0} + x_0 = 0$. For a given real number α, define

$$g(x) = \frac{3xe^x + 3x - \alpha e^x - \alpha x}{3(e^x + 1)}$$ for all real numbers X.

Then which one of the following statements is TRUE?

(A) For $\alpha = 2, \lim_{x \to x_0} \left| \frac{g(x) + e^{x_0}}{x - x_0} \right| = 0$

(B) For $\alpha = 2, \lim_{x \to x_0} \left| \frac{g(x) + e^{x_0}}{x - x_0} \right| = 1$

(C) For $\alpha = 3, \lim_{x \to x_0} \left| \frac{g(x) + e^{x_0}}{x - x_0} \right| = 0$

(D) For $\alpha = 3, \lim_{x \to x_0} \left| \frac{g(x) + e^{x_0}}{x - x_0} \right| = \frac{2}{3}$

Ans. **(C)**

Sol. $x_0 + e^{x_0} = 0$

So

$$g(x_0) = \frac{3x_0(-x_0) + 3x_0 - \alpha(-x_0) - \alpha(x_0)}{3(1 - x_0)} = \frac{3x_0(1 - x_0)}{3(1 - x_0)} = x_0$$

$$\lim_{x \to x_0} \frac{g(x) + e^{x_0}}{x - x_0} = \lim_{x \to x_0} \frac{g(x) - x_0}{x - x_0} = \lim_{x \to x_0} \frac{g(x) - g(x_0)}{x - x_0} = g'(x_0)$$

Now

$$3x - \alpha\left(\frac{e^x + x}{e^x + 1}\right) = g(x)$$

$$g'(x) = 3 - \alpha\left\{\frac{(e^x + 1)^2 - (e^x + x)e^x}{(e^x + 1)^2}\right\}$$

$$= 3 - \alpha + \frac{\alpha e^x (e^x + x)}{(e^x + 1)^2}$$

$$g'(x_0) = 3 - \alpha$$

If $\alpha = 3$, $g'(x_0) = 0 \Rightarrow$ (c)

2. Let $\mathbb{R}$ denote the set of all real numbers. Then the area of the region

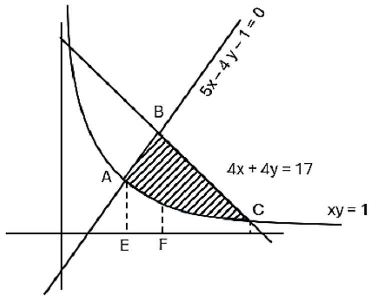
$$\left\{(x, y) \in \mathbb{R} \times \mathbb{R} : x > 0, y > \frac{1}{x}, 5x - 4y - 1 > 0, 4x + 4y - 17 < 0\right\}$$ is

(A) $\frac{17}{16} - \log_e 4$ (B) $\frac{33}{8} - \log_e 4$

(C) $\frac{57}{8} - \log_e 4$ (D) $\frac{17}{2} - \log_e 4$

Ans. **(B)**

Sol.



After solving equations we get point $B\left(2, \frac{9}{4}\right), A(1,1)$ and $C\left(4, \frac{1}{4}\right)$

Required area $= \frac{1}{2}\left(1 + \frac{9}{4}\right) \times 1 - \int_1^2 \frac{1}{x} dx + \frac{1}{2}\left(\left(\frac{9}{4} + \frac{1}{4}\right)\right) \times 2$

$-\int_2^4 \frac{1}{x} dx$

$= \frac{13}{8} + \frac{20}{8} - \ln 2 - (\ln 4 - \ln 2) = \frac{33}{8} - \ln 4$

3. The total number of real solutions of the equation

$\theta = \tan^{-1}(2\tan\theta) - \frac{1}{2}\sin^{-1}\left(\frac{6\tan\theta}{9 + \tan^2\theta}\right)$ is

(Here, the inverse trigonometric functions $\sin^{-1}x$ and $\tan^{-1}x$ assume values in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, respectively.)

(A) 1 (B) 2 (C) 3 (D) 5

Ans. **(C)**

Sol. $\theta = \tan^{-1}(2\tan\theta) - \frac{1}{2}\sin^{-1}\left(\frac{6\tan\theta}{9 + \tan^2\theta}\right)$

$= \tan^{-1}(2\tan\theta) - \frac{1}{2}\sin^{-1}\left(\frac{2\left(\frac{\tan\theta}{3}\right)}{1 + \left(\frac{\tan\theta}{3}\right)^2}\right)$, let $\frac{\tan\theta}{3} = \tan\alpha$

Case-1

$$\theta = \tan^{-1}(2\tan\theta) - \frac{1}{2}\sin^{-1}((\sin 2\alpha))$$

Therefore $\theta = \tan^{-1}(2\tan\theta) - \tan^{-1}\left(\frac{\tan\theta}{3}\right)$

$$\tan\theta = \frac{2\tan\theta - \frac{\tan\theta}{3}}{1 + \frac{2\tan^2\theta}{3}} = \frac{5\tan\theta}{3 + 2\tan^2\theta}$$

So $\tan\theta = 0$ or $\tan^2\theta = 1 \Rightarrow \tan\theta = 0, \pm 1$ then value of $\theta = 0, \frac{\pi}{4}$ and $-\frac{\pi}{4}$

Case-2

$$\theta = \tan^{-1}(2\tan\theta) - \frac{1}{2}\left(\pi - 2\tan^{-1}\left(\frac{\tan\theta}{3}\right)\right)$$

$$\theta = \tan^{-1}(2\tan\theta) - \frac{\pi}{2} + \tan^{-1}\left(\frac{\tan\theta}{3}\right)$$

$$\frac{\pi}{2} + \theta = \tan^{-1}(2\tan\theta) + \tan^{-1}\left(\frac{\tan\theta}{3}\right)$$

Form here $-\cot\theta = \frac{7\tan\theta}{3 - 2t^2} \Rightarrow -7\tan^2\theta = 3 - 2\tan^2\theta$

$-5\tan^2\theta = 3($ no real values of $\theta)$

Case-3

$$\theta = \tan^{-1}(2\tan\theta) - \frac{1}{2}\left(-\pi - 2\tan^{-1}\left(\frac{\tan\theta}{3}\right)\right)$$

$$\theta = \tan^{-1}(2\tan\theta) + \frac{\pi}{2} + \tan^{-1}\left(\frac{\tan\theta}{3}\right)$$

$$-\frac{\pi}{2} + \theta = \tan^{-1}(2\tan\theta) + \tan^{-1}\left(\frac{\tan\theta}{3}\right)$$

Form here $-\cot\theta = \frac{7\tan\theta}{3 - 2t^2} \Rightarrow -7\tan^2\theta = 3 - 2\tan^2\theta$

$-5\tan^2\theta = 3($ no real values of $\theta)$

Total no. of Solution $= 3$

4. Let S denote the locus of the point of intersection of the pair of lines

$4x-3y=12\alpha$

$4\alpha x+3\alpha y=12$

where α varies over the set of non-zero real numbers. Let T be the tangent to S passing through the points $(p,0)$ and $(0,q), q>0$, and parallel to the line $4x-\frac{3}{\sqrt{2}}y=0$.

(A) $-6\sqrt{2}$ (B) $-3\sqrt{2}$ (C) $-9\sqrt{2}$ (D) $-12\sqrt{2}$

Ans. **(A)**

Sol. $4x-3y=12\alpha$

$4x+3y=\frac{12}{\alpha}$

Multiplying

$16x^2-9y^2=144 \Rightarrow \frac{x^2}{9}-\frac{y^2}{16}=1$

Tangent is $y=mx\pm\sqrt{9m^2-16}$

Now $q>0 \Rightarrow y=mx+\sqrt{9m^2-16}$, where $m=\frac{4\sqrt{2}}{3}$

Hence $pq=-\left(\frac{\sqrt{9m^2-16}}{m}\right)\sqrt{9m^2-16}=-\frac{9m^2-16}{m}$

$=-3\left\{\frac{9\cdot\frac{32}{9}-16}{4\sqrt{2}}\right\}$

$-6\sqrt{2}$ Ans.

SECTION 2 (Maximum Marks: 12)

- This section contains FOUR (04) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s)
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 If only (all) the correct option(s) is (are) chosen.
  Partial Marks : +3 If all the four options are correct but ONLY three options are chosen.
  Partial Marks : +2 If three or more options are correct but ONLY two options are chosen and both of which are correct.
  Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).
  Negative Marks : -2 In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get + 2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get + 1 mark;
  choosing ONLY (B) will get + 1 mark;
  choosing ONLY (D) will get + 1 mark;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get -2 marks.

5. Let $I=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ and $P=\begin{pmatrix}2 & 0\\ 0 & 3\end{pmatrix}$. Let $Q=\begin{pmatrix}x & y\\ z & 4\end{pmatrix}$ for some non-zero real numbers x, y, and z, for which there is a 2×2 matrix R with all entries being non-zero real numbers, such that $QR = RP$. Then which of the following statements is (are) TRUE?

(A) The determinant of $Q-2I$ is zero
(B) The determinant of $Q-6I$ is 12
(C) The determinant of $Q-3I$ is 15
(D) $yz = 2$

Ans. **(AB)**

Sol. $QR = RP$

$QR-2R = RP-2R$

$(Q-2I)R = R(P-2I) = R\begin{pmatrix}0 & 1\\ 0 & 1\end{pmatrix}$

$\left|(Q-2')R\right| = |R|\left|P-2'\right| = 0$

$\because$ R is not null $\Rightarrow |Q-2/| = 0$

$QR = RP$

$QR-3R = RP-3R$

$\Rightarrow (Q-3/)R = R(P-3/) = R\begin{pmatrix}-1 & 0\\ 0 & 0\end{pmatrix}$

$|(Q-3I)R|=0$

$\therefore$ R is not null $\Rightarrow |Q-3I|=0$

$|Q-2I|=0 \Rightarrow 2(x-2)=yz \#(1)$

$|Q-3I|=0 \Rightarrow (x-3)=yz \#(2)$

Form (1) and (2) $2(x-2)=x-3$

$\Rightarrow x=1$

$\therefore yz=-2$

$|Q-6'|=\begin{vmatrix} x-6 & y \\ z & -2 \end{vmatrix}=2(x-6)-yz$

$=-2(1-6)+2$

$=12$

6. Let S denote the locus of the mid-points of those chords of the parabola $y^2=x$, such that the area of the region enclosed between the parabola and the chord is $\frac{4}{3}$. Let $\mathcal{R}$ denote the region lying in the first quadrant, enclosed by the parabola $y^2=x$, the curve S, and the lines $x=1$ and $x=4$. Then which of the following statements is (are) TRUE

(A) $(4,\sqrt{3})\in S$

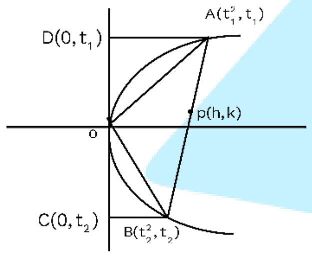
(B) $(5,\sqrt{2})\in S$

(C) Area of R is $\frac{14}{3}-2\sqrt{3}$

(D) Area of R is $\frac{14}{3}-\sqrt{3}$

Ans. **(C)**

Sol.



$2h=t_1^2+t_2^2 \Rightarrow t_1t_2=2k^2-h$

$2k=t_1+t_2$

Required Area $=\frac{1}{2}(t_1^2+t_2^2)(t_1-t_2)-\frac{1}{3}(t_1^3-t_2^3)=\frac{4}{3}$

$\Rightarrow 3(t_1^2+t_2^2)(t_1-t_2)-2(t_1^3-t_2^3)=8 \Rightarrow (t_1-t_2)\left[3(t_1^2+t_2^2)-2(t_1^2+t_2^2+t_1t_2)\right]=8$

$\Rightarrow (t_1-t_2)[t_1-t_2]^2=8 \Rightarrow (t_1-t_2)^2=t_1^2+t_2^2-2t_1t_2 =2h-(4k^2-2h)$

$4 \quad =4(h-k^2)$

Hence, S is $y^2=x-1 \Rightarrow$ Option (A) is correct

Now required area $=\int_1^4\left(\sqrt{x}-\sqrt{x-1}\right)dx \qquad =\frac{2}{3}\left\{x^{3/2}-(x-1)^{3/2}\right\}_1^4=\frac{2}{3}[(8-3\sqrt{3}-(1)]$

$=\frac{14}{3}-2\sqrt{3}$

7. Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two distinct points on the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ such that $y_1>0$, and $y_2>0$. Let C denote the circle $x^2+y^2=9$, and M be the point $(3,0)$.

Suppose the line $x=x_1$ intersects C at R, and the line $x=x_2$ intersects C at S, such that the y-coordinates of R and S are positive. Let $\angle ROM=\frac{\pi}{6}$ and $\angle SOM=\frac{\pi}{3}$, where O denotes the origin $(0,0)$. Let $|XY|$ denote the length of the line segment XY.

Then which of the following statements is (are) TRUE?

(A) The equation of the line joining P and Q is $2x+3y=3(1+\sqrt{3})$

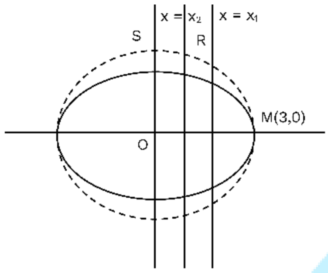
(B) The equation of the line joining P and Q is $2x+y=3(1+\sqrt{3})$

(C) If $N_2=(x_2,0)$, then $3|N_2Q|=2|N_2S|$

(D) If $N_1=(x_1,0)$, then $9|N_1P|=4|N_1R|$s

Ans. **(AC)**

Sol.



$\angle \text{ROM}=\frac{\pi}{6}$

$R\equiv\left(3\frac{\sqrt{3}}{2},\frac{3}{2}\right)$ $S\equiv\left(3\frac{1}{2},\frac{3\sqrt{3}}{2}\right)$ $P\equiv\left(3\frac{\sqrt{3}}{2},1\right)$ $\text{Q}\equiv\left(\frac{3}{2},\sqrt{3}\right)$

Equation of line PQ

$$(y-1)=\frac{\sqrt{3}-1}{\frac{3}{2}(1-\sqrt{3})}\left(x-3\frac{\sqrt{3}}{2}\right)$$

$$-\frac{3}{2}y+\frac{3}{2}=x-\frac{3\sqrt{3}}{2}$$

$$x+\frac{3}{2}y=\frac{3}{2}+\frac{3\sqrt{3}}{2}$$

$$2x+3y=3+3\sqrt{3}=3(1+\sqrt{3})\#(A)$$

(C) $\text{N}_2\equiv\left(\frac{3}{2},0\right)|\text{N}_2\text{Q}|=\sqrt{(\text{O}^2+3)}=\sqrt{3}$

(D) $|\text{N}_2\ \text{S}|=3\frac{\sqrt{3}}{2}3|\ \text{N}_2\text{Q}|=2|\ \text{N}_2\ \text{S}|\#(C)$

$N_1\equiv\left(\frac{3\sqrt{3}}{2},0\right)|N_1P|=1|N_1R|=\frac{3}{2}$

$9|N_1P|=9$, $4|N_1R|=6$ incorrect

8. Let $\mathbb{R}$ denote the set of all real numbers. Let $f:\mathbb{R}\to\mathbb{R}$ be defined by

$$f(x)=\begin{cases}\frac{6x+\sin x}{2x+\sin x} & \text{if } x\neq 0\\ \frac{7}{3} & \text{if } x=0\end{cases}$$

Then which of the following statements is (are) TRUE?

(A) The point $x=0$ is a point of local maxima of f

(B) The point $x=0$ is a point of local minima of f

(C) Number of points of local maxima of f in the interval $[\pi,6\pi]$ is 3

(D) Number of points of local minima of f in the interval $[2\pi,4\pi]$ is 1

Ans. **(B,C,D)**

Sol. $$f(x)=\begin{cases}\frac{6x+\sin x}{2x+\sin x} & \text{if } x=0\\ \frac{7}{3} & \text{if } x=0\end{cases}$$

$$f(0)=\frac{7}{3}\lim_{x\to 0}\frac{6+\frac{\sin x}{x}}{2+\frac{\sin x}{x}}=\left(\frac{7}{3}\right)^{+}$$

$X=0$ to local minima of $f(x)$.

$$f'(x)\frac{(6+\cos x)(2x+\sin x)-(2+\cos x)(6x+\sin x)}{(2x+\sin x)^2}$$

$$=\frac{-4x\cos x+4\sin x}{(2x+\sin x)^2}$$

$$=\frac{4(\cos x)(\tan x-x)}{(2x+\sin x)^2}$$

Tan $x=x$ has one solution b/w ($n\pi,(n+1)\pi$)

One maxima in $(2\pi,4\pi)$

3 Maximas in ($\pi,6\pi$)

SECTION 3 (Maximum Marks: 24)

- This section contains EIGHT (08) questions.
- The answer to each question is a NUMERICAL VALUE.
- For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, truncate/roundoff the value to TWO decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 ONLY if the correct numerical value is entered;
  Zero Marks : 0 In all other cases.

9. Let $y(x)$ be the solution of the differential equation

$$x^2\frac{dy}{dx}+xy=x^2+y^2,\ x>\frac{1}{e}$$

satisfying $y(1)=0$. Then the value of $2\frac{(y(e))^2}{y(e^2)}$ is

Ans. **0.37**

Sol. $$\frac{dy}{dx}=\frac{x^2+y^2-xy}{x^2}$$

$$y=vx$$

$$v+x\frac{dv}{dx}=1+v^2-v$$

$$\frac{dv}{(v-1)^2}=\frac{dx}{x}$$

$$-\frac{1}{v-1}=\ln x+c$$

$$\Rightarrow\frac{x}{x-y}=\ln x+c\Rightarrow\frac{1}{1-0}=c\Rightarrow c=1$$

$$\Rightarrow\frac{x}{x-y}=\ln x+1\Rightarrow x=x\ln x+x-y(\ln x+1)$$

$$\Rightarrow y=\frac{\ln x}{\ln x+1}$$

$$y(e)=\frac{1}{1+1}=\frac{1}{2},\ y(e^2)=\frac{2}{2+1}=\frac{2}{3}$$

$$\frac{(y(e))^2}{y(e^2)}=\frac{3}{4\times2}=\frac{3}{8}=0.375$$

Rounded off value $=0.37$

10. Let $a_0, a_1, \ldots, a_{23}$ be real numbers such that

$$\left(1+\frac{2}{5}x\right)^{23} = \sum_{i=0}^{23} a_i x^i$$

for every real number x. Let ar be the largest among the numbers aj for $0 \le j \le 23$.

Then the value of r is .

Ans. 6

Sol. Now $\frac{23+1}{1+\left|\frac{5}{2}\right|} = \frac{24\times 2}{7} = \frac{48}{7} = 6.857$

So largest coefficient is a_6 Hence ($r = 6$)

11. A factory has a total of three manufacturing units, M_1, M_2 and M_3, which produce bulbs independent of each other. The units $\mathrm{M}_1, \mathrm{M}_2$ and M_3 produce bulbs in the proportions of 2: 2: 1, respectively. It is known that 20% of the bulbs produced in the factory are defective. It is also known that, of all the bulbs produced by $\mathrm{M}_1, 15\%$ are defective. Suppose that, if a randomly chosen bulb produced in the factory is found to be defective, the probability that it was produced by M_2 is $\frac{2}{5}$.

If a bulb is chosen randomly from the bulbs produced by M_3, then the probability that it is defective is .

Ans. 0.30

Sol. D : Bulb is defective

M_i : Manufactured in unit i

$$P(D) = \sum_{i=1}^{3} P(M_i) P(D/M_i)$$

$$\frac{2}{10} = \frac{2}{5}\cdot\frac{15}{100} + \frac{2}{5}p_2 + \frac{1}{5}p_3 \Rightarrow \frac{10}{50} = \frac{3}{50} + \frac{2p_2}{5} + \frac{2p_3}{5}$$

$$\Rightarrow 20\mathrm{p}_2 + 10\mathrm{p}_2 = 7\#(1)$$

Here p_2 and p_3 are $P(D/M_2)$ and $P(D/M_3)$

Also $\frac{\frac{2}{5}p_2}{\frac{2}{10}} = \frac{2}{5} \Rightarrow p_2 = \frac{1}{5}$

Hence $p_3 = \frac{1}{10}\left(7 - 20\frac{1}{5}\right) = \frac{3}{10} \Rightarrow p_3 = 0.30$

12. Consider the vectors

$\vec{x} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{y} = 2\hat{i} + 3\hat{j} + \hat{k}$, and $\vec{z} = 3\hat{i} + \hat{j} + 2\hat{k}$

For two distinct positive real numbers α and β, define

$\vec{X} = \alpha\vec{x} + \beta\vec{y} - \vec{z}, \vec{Y} = \alpha\vec{y} + \beta\vec{z} - \vec{x}$ and $\vec{z} = \alpha\vec{z} + \beta\vec{x} - \vec{y}$

If the vectors $\vec{X}, \vec{Y}$ and $\vec{Z}$ lie in a plane, then the value of $\alpha + \beta - 3$ is .

Ans. **-2**

Sol. $$\left[\vec{X}\vec{Y}\vec{Z}\right] = \begin{vmatrix} \alpha & \beta & -1 \\ -1 & \alpha & \beta \\ \beta & -1 & \alpha \end{vmatrix}\begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{vmatrix} = -1$$

$$\Rightarrow \frac{(\alpha+\beta-1)}{2}\left\{(\alpha+1)^2 + (\beta+1)^2 + (\alpha-1)^2\right\} = 0$$

$$\Rightarrow \alpha + \beta - 1 = 0 \Rightarrow \alpha + \beta - 3 = -2 \text{ Ans.}$$

13. For a non-zero complex number z, let $\arg(z)$ denote the principal argument of z, with $-\pi < \arg(z) \leq$. Let ω be the cube root of unity for which $0 < \arg(\omega) < \pi$. Let

$$\alpha = \arg\left(\sum_{n=1}^{2025}(-\omega)^n\right)$$

Then the value $\frac{3\alpha}{\pi}$ of is .

Ans. **-2**

Sol. $$\arg\left(\frac{1+(-\omega)^{2025}}{1+\omega}\right) = \arg\left(\frac{2}{\omega}\right) = \arg\left(2\omega^2\right)$$

$$\alpha = \frac{-2\pi}{3}$$

then $\frac{3\alpha}{\pi}$ is equal to $3 \times \frac{\left(-\frac{2\pi}{3}\right)}{\pi} = -2$

14. Let $\mathbb{R}$ denote the set of all real numbers. Let $f:\mathbb{R}\to\mathbb{R}$ and $g:\mathbb{R}\to(0,4)$ be functions defined by $f(x)=\log_e(x^2+2x+4)$, and $g(x)=\frac{4}{1+e^{-2x}}$

Define the composite function $f\circ g^{-1}$ by $(f\circ g^{-1})(x)=f(g^{-1}(x))$, where g^{-1} is the inverse of the function g.

Then the value of the derivative of the composite function $f\circ g^{-1}$ at $x=2$ is ____.

Ans. **1**

Sol. Let $g^{-1}(x)=h(x)$

$$\frac{d}{dx}(f(h(x)))=f'(h(x))\cdot h'(x)$$

$f'(h(2)).\,h'(2)=$? as $h(2)=0$ then

$$f'(h(2)).\,h'(2)=f'(0).\,h'(2)$$

$$f'(x)=\frac{1}{x^2+2x+4}\times(2x+2)$$

$$f'(0)=\frac{2}{4}=\frac{1}{2}$$

For h'(2)

$$h(g(x)=x$$

$$h'(g(x))=\frac{1}{g'(x)},\text{ at } x=0$$

$$h'(2)=\frac{1}{g'(0)}$$

$$g'(x)=-\frac{4}{(1+e^{-x\times 2})^2}\times(-e^{-2x}\times 2)$$

$$g'(0)=\frac{8}{4}=2$$

So

$$f'(h(x)).h'(x)\ \text{ at }\ x=2\ \text{ is }=\frac{1}{2}\times 2=1$$

15. Let $\alpha = \frac{1}{\sin 60^\circ \sin 61^\circ} + \frac{1}{\sin 62^\circ \sin 63^\circ} + \cdots + \frac{1}{\sin 118^\circ \sin 119^\circ}$

Then the value of $\left(\frac{\operatorname{cosec} 1^0}{\alpha}\right)^2$ is .

Ans. 3

Sol. $\alpha = \frac{1}{\sin 1^\circ}\left[\frac{\sin\left(61^\circ - 60^\circ\right)}{\sin 60^\circ \sin 61^\circ} + \frac{\sin\left(63^\circ - 62^\circ\right)}{\sin 63^\circ \sin 62^\circ} + \ldots.. + \frac{\sin\left(119^\circ - 118^\circ\right)}{\sin 119^\circ \sin 118^\circ}\right]$

$\alpha \frac{1}{\sin 1^\circ}\left(\cot 60^\circ - \cot 61^\circ + \cot 62^\circ - \cot 63 + \cot 64^\circ \ldots\ldots\ldots\ldots\ldots \cot 118^\circ - \cot 119^\circ\right)$

$\cot\left(119^\circ\right) = -\cot\left(180^\circ - 61^\circ\right) = -\cot 61^\circ$

$\alpha = \frac{1}{\sin 1^\circ}\left(\cot 60^\circ\right) = \frac{1}{\sqrt{3}} \operatorname{cosec}^\circ$

$\left(\frac{\operatorname{cosec} 1^\circ}{\alpha}\right)^2 = \left(\frac{\operatorname{cosec} 1^\circ}{\frac{\operatorname{cosec} 1^\circ}{3}}\right) = 3$ Ans.

16. If $\alpha = \int_{\frac{1}{2}}^{2} \frac{\tan^{-1} x}{2x^2 - 3x + 2} dx$

then the value of $\sqrt{7} \tan\left(\frac{2\sqrt{7}}{\pi}\right)$ is .

(Here, the inverse trigonometric function $\tan^{-1} x$ assumes values in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.)

Ans. 21

Sol. $\alpha = \int_{1/2}^{2} \frac{\tan^{-1} x}{2x^2 - 3x + 2} dx$

Put $x = \frac{1}{t}$ then $dx = -\frac{1}{t^2} dt$

$\alpha = \int_{1/2}^{2} \frac{\tan^{-1}\left(\frac{1}{t}\right)}{\frac{2}{t^2} - \frac{3}{t} + 2} \cdot \frac{1}{t^2} dt$

$\alpha = \int_{1/2}^{2} \frac{\tan^{-1}\left(\frac{1}{t}\right)}{2t^2 - 3t + 2} dt \#(2)$

(1) $+(2)$

$\alpha = \frac{\pi}{4} \cdot \frac{1}{2} \int_{1/2}^{2} \frac{1}{\left(t^2 - \frac{3t}{2} + \frac{9}{16}\right) + \left(\frac{\sqrt{7}}{4}\right)^2} = \frac{\pi}{2\sqrt{7}} \tan^{-1}\left(3\sqrt{7}\right)$

$\sqrt{7} \tan\left(\frac{2\alpha\sqrt{7}}{\pi}\right) = \sqrt{7} \tan\left(\frac{2 \cdot \frac{\pi}{2\sqrt{7}} \tan^{-1}\left(3\sqrt{7}\right) \times \sqrt{7}}{\pi}\right)$

$= \sqrt{7} \times 3\sqrt{7} = 21$