PHYSICS

SECTION 1 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	: +3	If **ONLY** the correct option is chosen;
Zero Marks	: 0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	: -1	In all other cases.

1. A temperature difference can generate e.m.f. in some materials. Let S be the e.m.f. produced per unit temperature difference between the ends of a wire, σ the electrical conductivity and κ the thermal conductivity of the material of the wire. Taking M, L, T, I and K as dimensions of mass, length, time, current and temperature, respectively, the dimensional formula of the quantity $Z = \frac{S^2\sigma}{\kappa}$ is:

(A) $\left[M^0\ L^0\ T^0 I^0\ K^0\right]$

(B) $\left[M^0\ L^0\ T^0 I^0\ K^{-1}\right]$

(C) $\left[M^1\ L^2\ T^{-2} I^{-1}\ K^{-1}\right]$

(D) $\left[M^1\ L^2\ T^{-4} I^{-1}\ K^{-1}\right]$

Ans. **(B)**

Sol. $Z = \frac{S^2\sigma}{K}$

$S = \frac{\Delta V}{\Delta T}$ $\Delta V \Rightarrow$ Potential difference

$\Delta T \Rightarrow$ Temperature difference

$[S] = \frac{\left[ML^2\ T^{-3}I^{-1}\right]}{K} = ML^2\ T^{-3}I^{-1}\ K^{-1}$

$K \rightarrow$ Thermal conductivity

$[K] = MLT^{-3}\ K^{-1}$

$\sigma \rightarrow$ Electrical conductivity

$[\sigma] \rightarrow M^{-1}\ L^{-3}\ T^3I^2$

$[Z] = \frac{M^2L^4T^{-6}I^{-2}K^{-2} \times M^{-1}L^{-3}T^3I^2}{MLT^{-3}K^{-1}}$

$[Z] = M^0L^0T^0I^0K^{-1}$

2. Two co-axial conducting cylinders of same length ℓ with radii $\sqrt{2}R$ and $2R$ are kept, as shown in Fig. 1. The charge on the inner cylinder is Q and the outer cylinder is grounded. The annular region between the cylinders is filled with a material of dielectric constant $\kappa = 5$. Consider an imaginary plane of the same length ℓ at a distance R from the common axis of the cylinders. This plane is parallel to the axis of the cylinders. The cross-sectional view of this arrangement is shown in Fig. 2. Ignoring edge effects, the flux of the electric field through the plane is (ϵ_0 is the permittivity of free space):

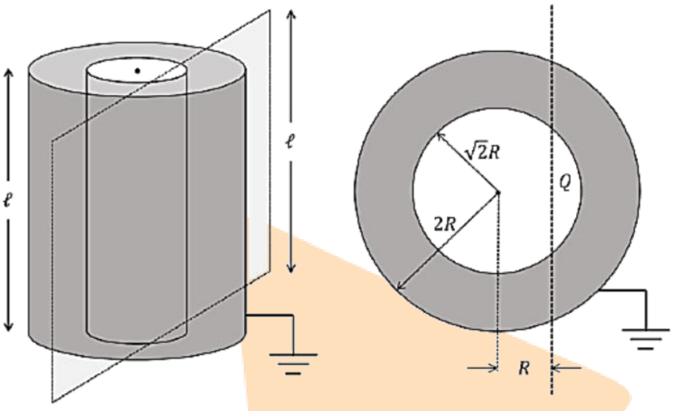
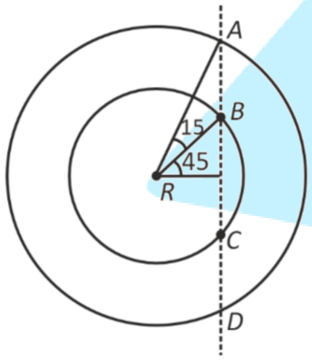


Fig. 1 Fig. 2

(A) $\frac{Q}{30\epsilon_0}$ (B) $\frac{Q}{15\epsilon_0}$ (C) $\frac{Q}{60\,\epsilon_0}$ (D) $\frac{Q}{120\epsilon_0}$

Ans. **(C)**



Sol.

$$\phi_{BC} = 0$$

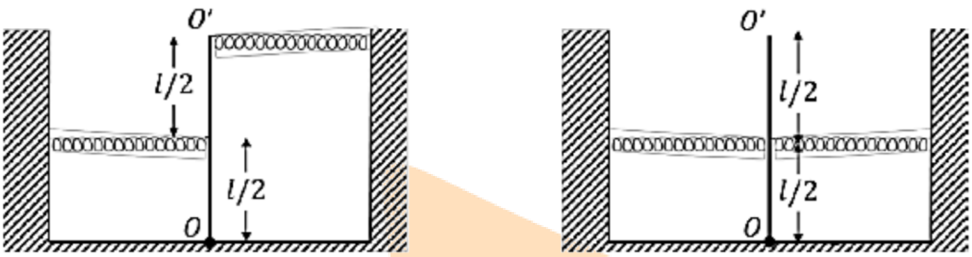
$$\phi_{AB} = \phi_{CD}$$

$$\phi_{AB} = \frac{15}{360} \times \frac{Q}{K\varepsilon_0}$$

$$= \frac{Q}{24K\varepsilon_0}$$

$$\phi_{\text{net}} = 2\phi_{AB} = \frac{Q}{12K\varepsilon_0} = \frac{Q}{60\varepsilon_0}$$

3. As shown in the figures, a uniform rod OO' of length l is hinged at the point O and held in place vertically between two walls using two massless springs of same spring constant. The springs are connected at the midpoint and at the top-end (O′) of the rod, as shown in Fig. 1 and the rod is made to oscillate by a small angular displacement. The frequency of oscillation of the rod is f_1. On the other hand, if both the springs are connected at the midpoint of the rod, as shown in Fig. 2 and the rod is made to oscillate by a small angular displacement, then the frequency of oscillation is f_2. Ignoring gravity and assuming motion only in the plane of the diagram, the value of $\frac{f_1}{f_2}$ is:



(A) 2 (B) $\sqrt{2}$ (C) $\sqrt{\frac{5}{2}}$ (D) $\sqrt{\frac{2}{5}}$

Ans. **(C)**

Sol. **Case I**

$$T_{net} = \left[K\frac{\ell}{2}\theta\right]\frac{\ell}{2} + K(\ell\theta)\times\ell$$

$$T_{net} = \left[\frac{5}{4}K\ell^2\right]\theta$$

$$T = 2\pi\sqrt{\frac{I}{K}} \Rightarrow f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{K}{I}}$$

Case-II

$$T_{net} = \left[\left[K\frac{\ell}{2}\theta\right]\frac{\ell}{2}\right]2 = \frac{K\ell^2}{2}\times\theta$$

$$\frac{f_1}{f_2} = \sqrt{\frac{K_1}{K_2}} = \sqrt{\frac{\frac{5}{4}K\ell^2}{\frac{K\ell^2}{2}}} = \sqrt{\frac{10}{4}} = \sqrt{\frac{5}{2}}$$

4. Consider a star of mass $m_2 kg$ revolving in a circular orbit around another star of mass $m_1 kg$ with $m_1 \gg m_2$. The heavier star slowly acquires mass from the lighter star at a constant rate of γ kg / s. In this transfer process, there is no other loss of mass. If the separation between the centers of the stars is r, then its relative rate of change $\frac{1}{r}\frac{dr}{dt}$ (in s^{-1}) is given by:

(A) $-\frac{3\gamma}{2m_2}$ (B) $-\frac{2\gamma}{m_2}$ (C) $-\frac{2\gamma}{m_1}$ (D) $-\frac{3\gamma}{2m_1}$

Ans. **(B)**

Sol. $\frac{C_1 m_1 m_2}{r^2} = \frac{m_2 v^2}{r} = C_1 m_1 m_2 = m_2 r v^2$

$\Rightarrow C_1 m_1 m_1 = \frac{m_2^2 r^2 v^2}{m_2 r}$

$C_1 m_1 m_2^2 r = (m_2 v r)^2$

$C_1 m_1 m_2^2 r = L^2$

$\Rightarrow \mathrm{m_1\ m_2^2 r = cons\ tan\ t}$

Differential

$(m_1 m_2^2)\frac{dr}{dt} + r\left[\frac{dm_1}{dt} m_2^2 + m_1^2 m_2 \frac{dm_2}{dt}\right] = 0$

$m_1 m_2 \left(\frac{dr}{dt}\right) + r\left[m_2\left[m_2 \frac{dm_1}{dt} + 2m_1 \frac{dm_2}{dt}\right]\right] = 0$

$m_1 m_2 \left(\frac{dr}{dt}\right) + r\left[m_2 (r) + 2m_1 (-r)\right] = 0$

$\frac{dr}{dt} = \frac{rv(2m_1 - m_2)}{m_1 m_2}$

$\frac{1}{r}\frac{dr}{dt} = r\frac{2m_1}{m_1 m_2} = -\frac{2r}{m_2}$

SECTION 2 (Maximum Marks: 12)

- This section contains **THREE (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	: +4	**ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks	: +3	If all the four options are correct but **ONLY** three options are chosen;
Partial Marks	: +2	If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks	: +1	If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks	: 0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	: -2	In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY(A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and choosing any other combination of options will get -2 marks.

5. A positive point charge of 10^{-8}C is kept at a distance of 20 cm from the center of a neutral conducting sphere of radius 10 cm . The sphere is then grounded and the charge on the sphere is measured. The grounding is then removed and subsequently the point charge is moved by a distance of 10 cm further away from the center of the sphere along the radial direction. Taking $\frac{1}{4\pi\epsilon_0} = 9\times10^9\,\text{Nm}^2/\text{C}^2$ (where ϵ_0 is the permittivity of free space), which of the following statements is/are correct:

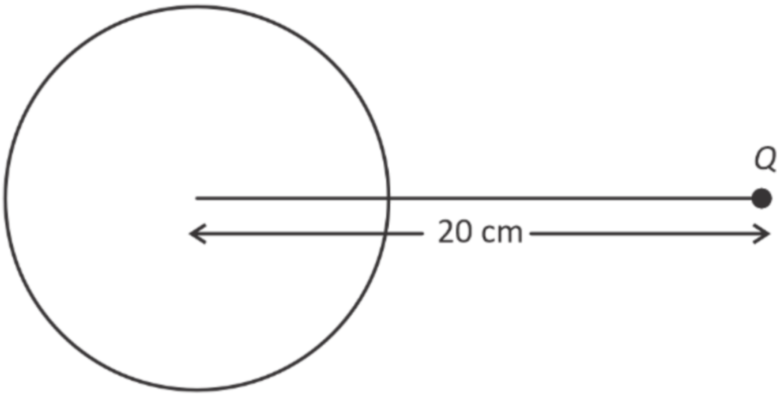
(A) Before the grounding, the electrostatic potential of the sphere is 450 V.

(B) Charge flowing from the sphere to the ground because of grounding is $5\times10^{-9}\text{c}$.

(C) After the grounding is removed, the charge on the sphere is $-5\times10^{-9}\text{C}$.

(D) The final electrostatic potential of the sphere is 300 V.

Ans. **(A, B, C)**



Sol.

(A) $V = \frac{KQ}{0.2} = \frac{9\times10^{9}\times10^{-8}}{0.2} = 90\times5 = 450\ \text{V}$

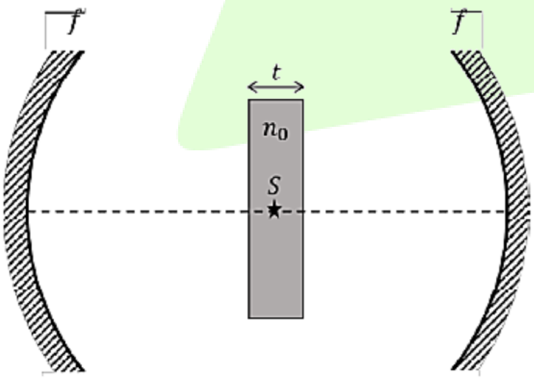
(B) $\frac{KQ}{0.2} = \frac{Kq}{0.1} \Rightarrow q = \frac{Q}{2} = \frac{10^{-8}}{2} = 5\times10^{-9}\,\text{C}$

(D) $\frac{KQ}{0.3} + \frac{K\left(\frac{-Q}{2}\right)}{0.1} = KQ\left(\frac{1}{0.3} - \frac{3}{0.3}\right)$

$\Rightarrow -2\left(\frac{KQ}{0.3}\right) = \frac{-2\times9\times10^{9}\times10^{-8}}{3\times10^{-1}}$

$= -600\ \text{V}$

6. Two identical concave mirrors each of focal length f are facing each other as shown in the schematic diagram. The focal length f is much larger than the size of the mirrors. A glass slab of thickness t and refractive index n_0 is kept equidistant from the mirrors and perpendicular to their common principal axis. A monochromatic point light source S is embedded at the center of the slab on the principal axis, as shown in the schematic diagram. For the image to be formed on S itself, which of the following distances between the two mirrors is/are correct:



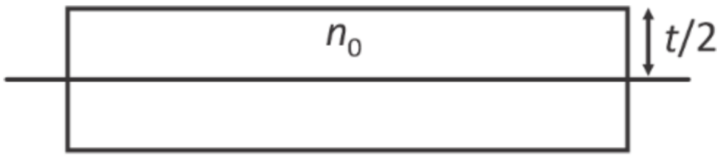
(A) $4f + \left(1 - \frac{1}{n_0}\right)t$

(B) $2f + \left(1 - \frac{1}{n_0}\right)t$

(C) $4f + (n_0 - 1)t$

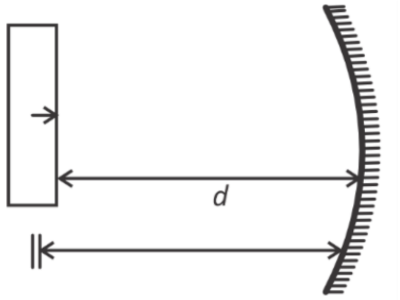
(D) $2f + (n_0 - 1)t$

Ans. **(A, B)**



Sol.

$h' = \frac{h_0}{\mu_0}$



$\Rightarrow d + \frac{t}{2n_0} = 2f$

$d = 2f - \frac{t}{2n_0}$

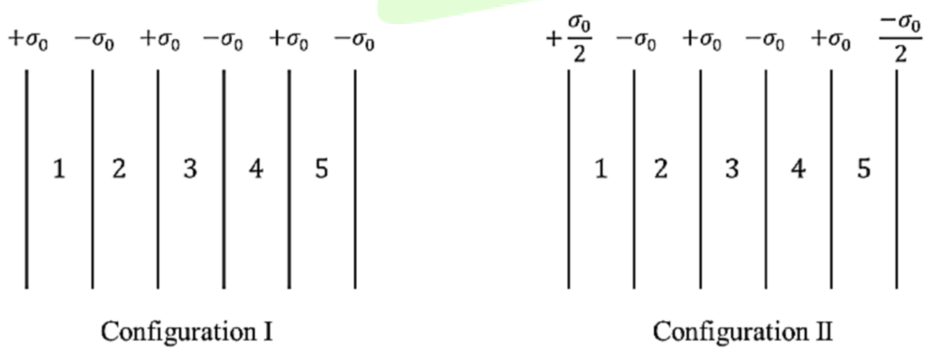
$d + \frac{t}{2} = 2f - \frac{t}{2n_0} + \frac{t}{2}$

$2\left(d + \frac{t}{2}\right) = 4f + \frac{t}{2}\left(1 - \frac{1}{n_0}\right)$

$= 4f + t\left(1 - \frac{1}{n_0}\right)$

7. Six infinitely large and thin non-conducting sheets are fixed in configurations I and II. As shown in the figure, the sheets carry uniform surface charge densities which are indicated in terms of σ_0. The separation between any two consecutive sheets is 1μ m. The various regions between the sheets are denoted as $1, 2, 3, 4$ and 5. If $\sigma_0 = 9\mu C/m^2$, then which of the following statements is/are correct:

(Take permittivity of free space $\epsilon_0 = 9\times10^{-12}$ F/m)



(A) In region 4 of the configuration I, the magnitude of the electric field is zero.

(B) In region 3 of the configuration II, the magnitude of the electric field is $\frac{\sigma_0}{\epsilon_0}$.

(C) Potential difference between the first and the last sheets of the configuration I is 5 V.

(D) Potential difference between the first and the last sheets of the configuration II is zero.

Ans. **(A)**

Sol. (A) $\frac{\sigma_0}{2\varepsilon_0}-\frac{\sigma_0}{2\varepsilon_0}+\frac{\sigma_0}{2\varepsilon_0}-\frac{\sigma_0}{2\varepsilon_0}-\frac{\sigma_0}{2\varepsilon_0}+\frac{\sigma_0}{2\varepsilon_0}=0$

(B) $\frac{\sigma_0}{4\varepsilon_0}-\frac{\sigma_0}{2\varepsilon_0}+\frac{\sigma_0}{2\varepsilon_0}+\frac{\sigma_0}{2\varepsilon_0}-\frac{\sigma_0}{2\varepsilon_0}+\frac{\sigma_0/2}{2\varepsilon_0}$

$\frac{\sigma_0}{4\varepsilon_0}+\frac{\sigma_0}{4\varepsilon_0}=\frac{\sigma_0}{2\varepsilon_0}$

(C) $E_1\frac{\sigma_0-\sigma_0-\sigma_0+\sigma_0-\sigma_0+\sigma_0}{2\varepsilon_0}=\frac{\sigma_0}{\varepsilon_0}$

$E_2\frac{\sigma_0-\sigma_0-\sigma_0+\sigma_0-\sigma_0+\sigma_0}{2\varepsilon_0}=0$

$E_3\frac{\sigma_0-\sigma_0+\sigma_0+\sigma_0-\sigma_0+\sigma_0}{2\varepsilon_0}=\frac{\sigma_0}{\varepsilon_0}$

$E_4\frac{\sigma_0-\sigma_0+\sigma_0-\sigma_0-\sigma_0+\sigma_0}{2\varepsilon_0}=0$

$E_5\frac{\sigma_0-\sigma_0+\sigma_0-\sigma_0+\sigma_0+\sigma_0}{2\varepsilon_0}=\frac{\sigma_0}{\varepsilon_0}$

$\Delta V=\left(\frac{\sigma_0}{\varepsilon_0}\right)\left(10^{-6}\right)+\frac{\sigma_0}{\varepsilon_0}\left(10^{-6}\right)+\frac{\sigma_0}{\varepsilon_0}\left(10^{-6}\right)$

$=\left[\frac{9\times10^{-6}}{9\times10^{-12}}\times10^{-6}\right]^3=3\text{volt}$

(D) $E_1=\frac{\frac{\sigma_0}{2}+\sigma_0-\sigma_0+\sigma_0-\sigma_0+\sigma_0/2}{2\varepsilon_0}=\frac{\sigma_0}{2\varepsilon_0}$

$E_2=\frac{\frac{\sigma_0}{2}-\sigma_0-\sigma_0+\sigma_0-\sigma_0+\sigma_0/2}{2\varepsilon_0}=-\frac{\sigma_0}{2\varepsilon_0}$

$E_3=\frac{\frac{\sigma_0}{2}-\sigma_0+\sigma_0+\sigma_0-\sigma_0+\sigma_0/2}{2\varepsilon_0}=\frac{\sigma_0}{2\varepsilon_0}$

$E_4=\frac{\frac{\sigma_0}{2}-\sigma_0+\sigma_0-\sigma_0-\sigma_0+\sigma_0/2}{2\varepsilon_0}=-\frac{\sigma_0}{2\varepsilon_0}$

$$E_5 = \frac{\frac{\sigma_0}{2} - \sigma_0 + \sigma_0 - \sigma_0 + \sigma_0 + \sigma_0 / 2}{2\varepsilon_0} = \frac{\sigma_0}{2\varepsilon_0}$$

$$\Delta V = \frac{\sigma_0}{2\varepsilon_0} \times d = \frac{1}{2} \times \frac{9\times10^{-6}}{9\times10^{-12}} \times 10^{-6} = 0.5\text{volt}$$

8. The efficiency of a Carnot engine operating with a hot reservoir kept at a temperature of 1000 K is 0.4. It extracts 150 J of heat per cycle from the hot reservoir. The work extracted from this engine is being fully used to run a heat pump which has a coefficient of performance 10. The hot reservoir of the heat pump is at a temperature of 300 K. Which of the following statements is/are correct:

(A) Work extracted from the Carnot engine in one cycle is 60 J.

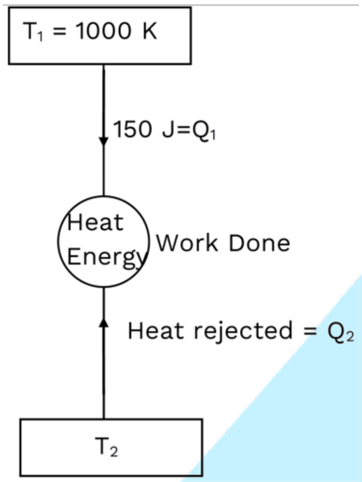
(B) Temperature of the cold reservoir of the Carnot engine is 600 K.

(C) Temperature of the cold reservoir of the heat pump is 270 K.

(D) Heat supplied to the hot reservoir of the heat pump in one cycle is 540 J.

Ans. **(A, B, C)**

Sol.



$$\eta = 0.4 = 1 - \frac{T_2}{T_1}$$

$$0.4 = 1 - \frac{T_2}{1000}$$

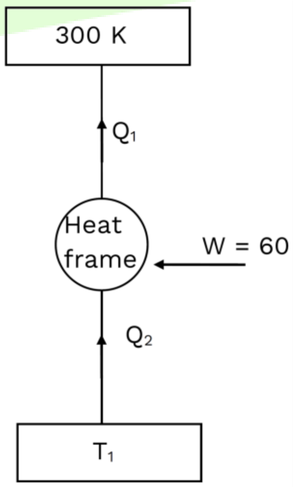
$$0.6 = \frac{T_2}{1000} \Rightarrow T_2 = 600 \text{ K}$$

$$\eta = \frac{W}{Q_1} = 0.4$$

$$10 = \frac{Q_2}{60}$$

$$\text{COP} = \frac{300}{300 - T'} = 10$$

$$T' = 270 \text{ K}$$



$$\text{COP} = 10 = \frac{Q_2}{W}$$

$$Q_2 = 600J, Q_1 = 60 + Q_2$$

SECTION 3 (Maximum Marks: 24)

- This section contains SIX (08) questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/roundoff** the value to TWO decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : **+4 ONLY** if the correct numerical value is entered;

Zero Marks : 0 In all other cases.

9. A conducting solid sphere of radius R and mass M carries a charge Q. The sphere is rotating about an axis passing through its center with a uniform angular speed ω. The ratio of the magnitudes of the magnetic dipole moment to the angular momentum about the same axis is given as $\alpha \frac{Q}{2M}$. The value of α is ______ .

Ans. **(1.67)**

Sol. $\frac{M}{L} = \frac{q}{2m}$

$M_{shell} = M_{conducting\ shell}$

$M_{CS} = \left(\frac{q}{2m}\right)\frac{2}{3}mr^2\omega$

$L_{SS} = \frac{2}{5}mr^2\omega$

$\frac{M}{L} = \frac{q}{2m}\frac{2}{3}\times\frac{5}{2}$

$= \left(\frac{q}{2m}\right)\frac{5}{3}$

$\alpha = 01.67$

10. A hydrogen atom, initially at rest in its ground state, absorbs a photon of frequency v_1 and ejects the electron with a kinetic energy of 10 eV. The electron then combines with a positron at rest to form a positronium atom in its ground state and simultaneously emits a photon of frequency v_2. The center of mass of the resulting positronium atom moves with a kinetic energy of 5 eV. It is given that positron has the same mass as that of electron and the positronium atom can be considered as a Bohr atom, in which the electron and the positron orbit around their center of mass. Considering no other energy loss during the whole process, the difference between the two photon energies (in eV) is _______ .

Ans. **(11.80)**

Sol. $E_1 = E_{ionisation} + KE_e$

$= 13.6 + 10 = 23.6\text{eV}$

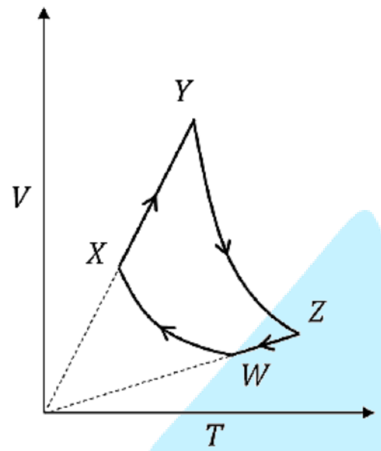
$10\text{eV} = (\text{KE})_{\text{com}} + \text{ Ground State energy} + \text{hv}_2$

$10\text{eV} = 5\text{eV} - 6.8\text{eV} + \text{hv}_2$

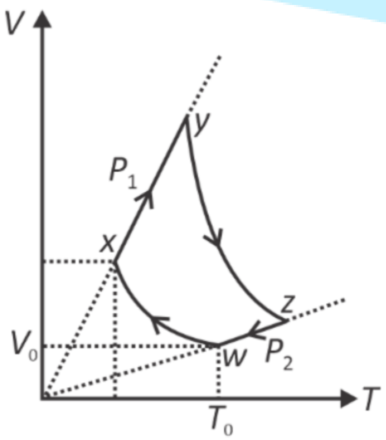
$E_2 = \text{hv}_2 = 11.8\text{eV}$

$E_1 - E_2 = 11.80\text{eV}$

11. An ideal monatomic gas of n moles is taken through a cycle WXYZW consisting of consecutive adiabatic and isobaric quasi-static processes, as shown in the schematic V-T diagram. The volume of the gas at W, X and Y points are, $64\text{ cm}^3, 125\text{ cm}^3$ and 250 cm^3, respectively. If the absolute temperature of the gas T_w at the point W is such that $nRT_w = 1J$ (R is the universal gas constant), then the amount of heat absorbed (in J) by the gas along the path XY is ______ .



Ans. **(01.60)**



Sol.

$$Q_{xy} = P_1\left(V_y - V_x\right) + nR\left(T_y - T_x\right)\frac{3}{2}$$

$$Q_{xy} = \frac{5}{2}R\left(T_y - T_x\right)n$$

$$T_w \cdot (64)\frac{2}{3} = T_x(125)\frac{2}{3}$$

$$T_w \cdot 16 = T_x \cdot 25$$

$$T_x = \frac{16}{25}T_w$$

$$\frac{T_y}{V_y} = \frac{T_x}{V_x}$$

$$T_y = T_x\left(\frac{250}{125}\right) = 2T_x$$

$$Q_{xy} = \frac{5}{2}nR(T_x)$$

$$= \frac{5}{2}nR \cdot \frac{16}{25}T_w$$

$$= \frac{8}{5}$$

$$Q_{xy} = 1.6$$

12. A geostationary satellite above the equator is orbiting around the earth at a fixed distance r_1 from the center of the earth. A second satellite is orbiting in the equatorial plane in the opposite direction to the earth's rotation, at a distance r_2 from the center of the earth, such that $r_1 = 1.21 r_2$. The time period of the second satellite as measured from the geostationary satellite is $\frac{24}{p}$ hours. The value of p is _____ .

Ans. **(02.33)**

Sol. $$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3}$$

$$\frac{T_1^2}{T_2^2} = (1.21)^3$$

$r_1 =$ geostationary

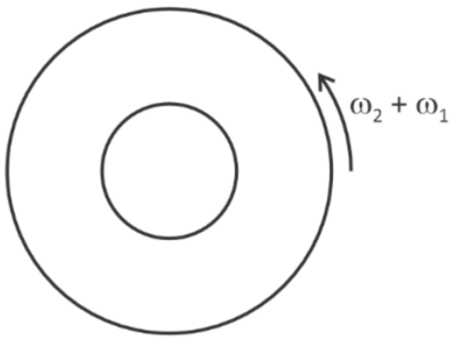
$$r_2 = \frac{r_1}{1.21}$$

$$\frac{T_1^2}{T_2^2} = (1.1)^6$$

$$\frac{T_1}{T_2} = (1.1)^3$$

$$\frac{T_1}{T_2} = 1.331$$

$$\omega_2 = (1.331)\,\omega_1$$

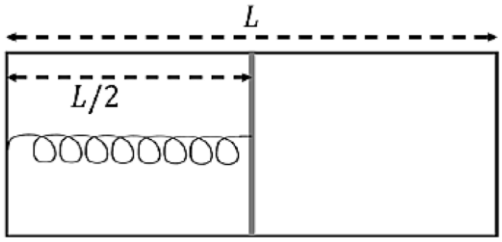


$$T_{\text{measured}} = \frac{2\pi}{\omega_2 + \omega_1} = \frac{2\pi}{2.331\omega_1}$$

$$= \frac{24}{2.331} = \frac{24\times 3}{7}$$

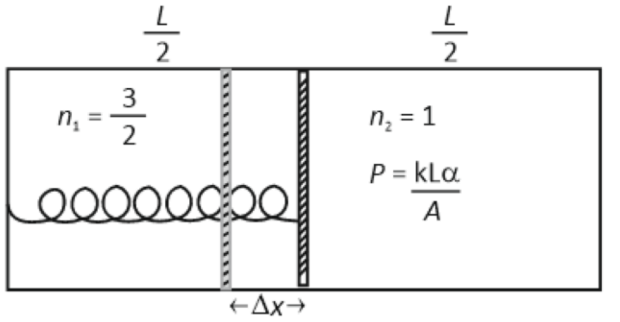
$$= 2.33$$

13. The left and right compartments of a thermally isolated container of length L are separated by a thermally conducting, movable piston of area A. The left and right compartments are filled with $\frac{3}{2}$ and 1 moles of an ideal gas, respectively. In the left compartment the piston is attached by a spring with spring constant k and natural length $\frac{2L}{5}$. In thermodynamic equilibrium, the piston is at a distance $\frac{L}{2}$ from the left and right edges of the container as shown in the figure. Under the above conditions, if the pressure in the right compartment is $P = \frac{kL}{A}\alpha$, then the value of α is ______ .



Ans. **(00.20)**

Sol.

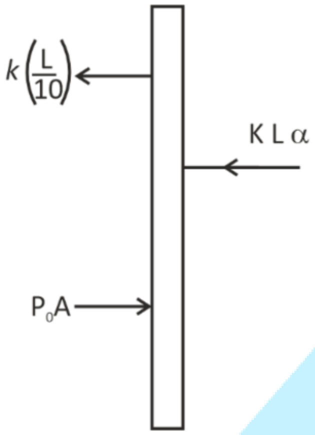


$$\Delta x = \frac{L}{2} - \frac{2L}{5} = \frac{L}{10},$$

$$pv = nRT$$

$$P_0 = \frac{3}{2}\frac{kl\alpha}{A}$$

On piston



$$P_0A = \frac{3}{2}KL\alpha$$

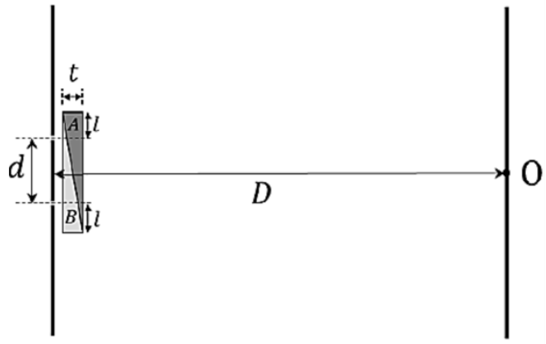
At equilibrium

$$\frac{KL}{10} - \frac{3}{2}KL\alpha = -KL\alpha$$

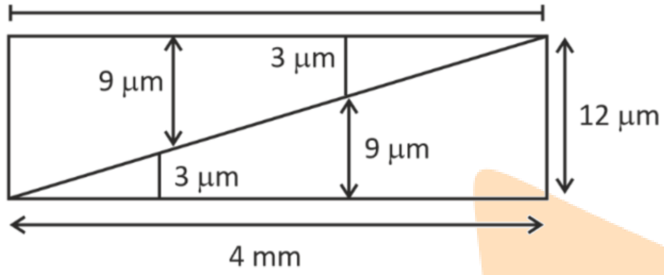
$$\alpha = \frac{2}{10} = 0.2$$

$$\Rightarrow 00.20$$

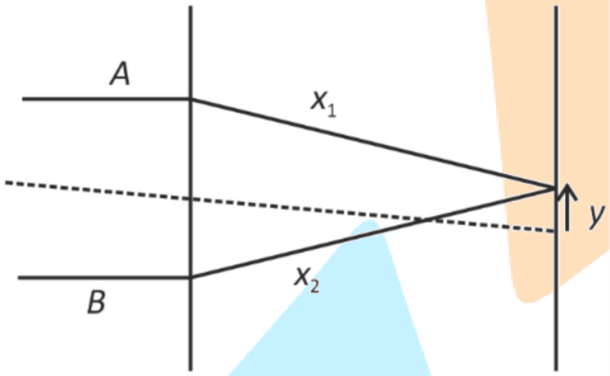
14. In a Young's double slit experiment, a combination of two glass wedges A and B, having refractive indices 1.7 and 1.5, respectively, are placed in front of the slits, as shown in the figure. The separation between the slits is $d = 2$ mm and the shortest distance between the slits and the screen is $D = 2$ m. Thickness of the combination of the wedges is $t = 12\mu$ m. The value of l as shown in the figure is 1 mm. Neglect any refraction effect at the slanted interface of the wedges. Due to the combination of the wedges, the central maximum shifts (in mm) with respect to O by ______ .



Ans. (01.20)



Sol.



$(x_1)_{\text{extra}} = \left[(1.5-1)^3 + (1.7-1)9\right]\times 10^{-6}$

$=(1.5+6.3)\times 10^{-6}$

$=7.8\times 10^{-6}$

$(x_2)_{\text{extra}} = \left[(1.5-1)9 + (1.7-1)3\right]\times 10^{-6}$

$=(4.5+2.1)\times 10^{-6}$

$=6.6\times 10^{-6}$

$\Delta x = 1.2\times 10^{-6}$

$\frac{d}{D}y = 1.2\times 10^{-6}$

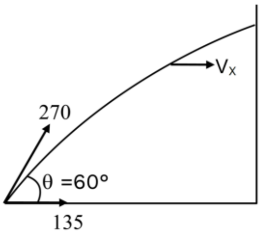
$y = \frac{1.2\times 10^{-6}\times 2}{2\times 10^{-3}}$

$=1.2$ mm

15. A projectile of mass 200 g is launched in a viscous medium at an angle 60° with the horizontal, with an initial velocity of $270\ \text{m/s}$. It experiences a viscous drag force $\vec{F} = -c\vec{v}$ where the drag coefficient $c = 0.1\ \text{kg/s}$ and $\vec{v}$ is the instantaneous velocity of the projectile. The projectile hits a vertical wall after 2 s. Taking $e = 2.7$, the horizontal distance of the wall from the point of projection (in m) is

Ans. **(170)**

Sol.



$F_x = -CV_x = \text{mass}$

$a_x = -\frac{C}{m}v_x = v_x\frac{dv_x}{dx}$

$dV_x = -\frac{C}{m}dx$

$135 - v_x = -\frac{C}{m}x$

$v_x = 135 - \frac{C}{m}x$

$\frac{dx}{dt} = 135 - \frac{C}{m}x$

$\frac{dx}{135 - \frac{C}{m}V_x} = dt$

$\frac{1}{-c/m}\ln\left(135 - \frac{c}{m}x\right) = t$

$\left[\ell m\left(135 - \frac{C}{m}x\right)\right]_0^x = -\frac{Ct}{m}$

$\ell m\left(135 - \frac{C}{m}x\right) - \ell n(135) = -\frac{0.1}{0.2}\times 2 = -1$

$\ln\left(\frac{135}{135 - \frac{C}{m}x}\right) = 1$

$\frac{135}{135 - \frac{C}{m}x} = e$

$135 = e[135 - 0.5x]$

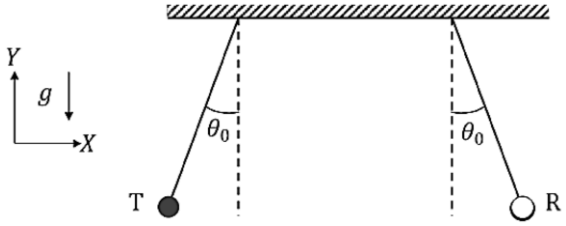
$\frac{135}{e} = 135 - 0.5x$

$0.5x = 135 - \frac{135}{e}$ $\quad x = \frac{135}{0.5}\left(1 - \frac{1}{e}\right)$ $\quad = 270\left(\frac{2.7-1}{2.7}\right)$ $\quad = 100\times 1.7$

$x = 170$

16. An audio transmitter (T) and a receiver (R) are hung vertically from two identical massless strings of length 8 m with their pivots well separated along the X axis. They are pulled from the equilibrium position in opposite directions along the X axis by a small angular amplitude

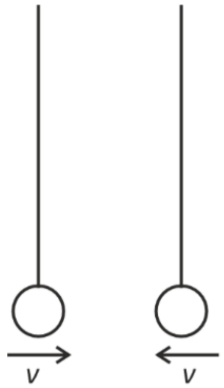
$\theta_0 = \cos^{-1}(0.9)$ and released simultaneously. If the natural frequency of the transmitter is 660 Hz and the speed of sound in air is 330 m/s, the maximum variation in the frequency (in Hz) as measured by the receiver (Take the acceleration due to gravity $g = 10 \text{ m/s}^2$) is _______.



Ans. **(32.00)**

Sol. $L = 8 \text{ m} \theta_0 = \cos^{-1}(0.9) f_0 = 660 \text{ Hz}$

$V_s = 330 \text{ g} = 10 \text{ m/s}^2$



$$v = \sqrt{2gl(1-\cos\theta)}$$

$$= \sqrt{2\times10\times8\times0.1} = 4 \text{ m/s}$$

$$f_{max} = f_0\left(\frac{v_s + v}{v_s - v}\right)$$

$$f_{min} = f_0\left(\frac{v_s - v}{v_s + v}\right)$$

$$f_{max} - f_{min} = f_0\left(\frac{v_s + v}{v_s - v} - \frac{v_s - v}{v_s + v}\right)$$

$$= f_0\left(\frac{4\times v_s \times v}{\left(v_s^2 - v^2\right)}\right) = \frac{660\times4\times330\times4}{\left(330^2 - 4^2\right)} \approx 32 \text{ Hz}$$

$$\Delta f = 32 \text{ Hz}$$